New Methods of Characterizing Traveling Ionospheric Disturbances using GNSS Measurements

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Outline

- Hooke TID Model
  - Hooke TEC analytical derivation
  - Analysis of satellite motion distortion
- Issues with standard GNSS TID estimation methods
- New spectral methods
  - Simulation
  - Actual Data
- General sensitivity of results to satellite motion
- Summary
Derivation of Hooke AGW TID Model

- Electron density TID due to AGW
- Ad-hoc saturation and decay with altitude
- Ad-hoc horizontal windowing function
- Vertical wavelength obtained from dispersion relation
- Parameters required: $k_x, k_y, f, u_b$ at reference altitude, phase, vertical neutral scale height

$$\delta N(z \leq z_T) = Re \left\{ \left( \frac{u_b \sin(I)}{\omega} \right) \frac{N(z)}{L_N(z)} e^{\frac{(z-z_*)}{2H}} e^{i\left[ \omega(t-t_0) - \vec{k} \cdot \vec{r} + \frac{\pi}{2} + \phi_0 - \Phi_N(z) \right]} \right\} W_S(\rho, t)$$

$$\delta N(z > z_T) = Re \left\{ \left( \frac{u_b \sin(I)}{\omega} \right) \frac{N(z)}{L_N(z)} e^{\frac{-(z-z_T)}{2H_T}} e^{i\left[ \omega(t-t_0) - \vec{k} \cdot \vec{r} + \frac{\pi}{2} + \phi_0 - \Phi_N(z) \right]} \right\} W_S(\rho, t)$$

$$\frac{1}{L_N(z)} = \left[ \left( \frac{1}{H_N(z)} + \frac{1}{2H} \right)^2 + \frac{k_{br}^2}{\sin^2(I)} \right]^{\frac{1}{2}}$$

$$\Phi_N(z) = \tan^{-1} \left[ \frac{k_{br}}{\sin(I)} \left( \frac{1}{H_N(z)} + \frac{1}{2H} \right)^{-1} \right]$$
Derivation of Hooke TEC – with satellite distortion

\[ \delta T(x_r, y_r, \theta_r(t), \alpha_r(t), t) = \int ds \delta N \]

\[ x = x_r + \cot(\theta_r) \sin(\alpha_r) z \]
\[ y = y_r + \cot(\theta_r) \cos(\alpha_r) z \]
\[ z = \sin(\theta_r) S \]

Slant integral of TID gives perturbed TEC:

- Approximate geometry as local Cartesian coordinates at “center of wave
- Use location of station, elevation and azimuth to go from slant to vertical integration
- Note: ignore horizontal gradients in background density

\[ \delta T(x_r, y_r, \theta_r(t), \alpha_r(t), t) = \left( \frac{u_b \sin(I)}{\omega} \right) Re \left[ \langle N(t) \rangle e^{i(\omega(t-t_0)-k_x x_r - k_y y_r + \frac{\pi}{2} - \phi_0)} \right] W_S(\rho, t) \]

\[ \langle N \rangle = \frac{1}{\sin(\theta_r)} \int dz \frac{N(z)}{L_N(z)} E(z) e^{-i[(k_x \sin(\alpha_r) + k_y \cos(\alpha_r)) \cot(\theta_r) + k_z] z + \Phi_N(z)} \]

\[ E(z) = e^{-\frac{(z-z_*)}{2H}} (z \leq z_T) = e^{-\frac{(z-z_T)}{2H}} (z > z_T) \]

- Effect of satellite motion
- Multiplied by altitude
- Thus altitude distribution very important

- No simple thin shell
GPS Station 1LSU
• Actual elevations and azimuths from one PRN over 6 hours
• Ground station located at 300km east from central location and 200km north from central location
• Blue curves are what a ground station would see from fixed non moving satellite.
• Red curve is the motion only due to the satellite
• Black curve is what the GPS receiver would see.

Static Bkgd w/ 1000 km horizontal scales
Previous Methods of Estimating TIDS from GNSS

- Closely clustered sets of receivers
- Receiver distances < wavelength of TID
  - Don’t know wavelengths ahead of time, cannot always guarantee
- Need to choose ionospheric pierce point altitude
  - Separation between receivers
  - Velocity of pierce point
  - Period very sensitive to pierce point altitude and IPP velocity
- Fundamental problem is TIDs are not thin shells.
  - Extended in altitude over 100+ km
  - Thus, velocity, period very sensitive to altitude effects
  - Cannot use one shell
  - Particularly case when speeds are close to GPS IPP speeds
A new technique: Spectral Methods

- Use as many GPS receivers as possible over as many different possible baselines pairs
  - Range from very small separation to largest as possible separation
  - Still need high elevation angles
- Cross correlation estimator
  - For each PRN take all baselines between pairs of receivers. For example, 60 receivers gives 1770 baselines
  - Compute cross correlations for each baseline pair
    \[ C_{i,j}(\tau, \Delta \rho_{i,j}) \approx \cos \left( (\omega - \vec{k} \cdot \vec{v}_j) \tau - \vec{k} \cdot \Delta \rho_{i,j} \right) \]
- Spectrum estimator
  - Take the zero time delay – this avoids any reference to frequency or velocity since:
  - Then the Spectrum is Fourier Transform of the spatial correlation (the above with zero time delay)
- When we have 100s – 1000s of correlation pairs, we can approximate that integral as a sum over all correlations:
  \[ S(k_x, k_y) \approx \sum_n C(\Delta \rho_n) e^{i(\vec{k} \cdot \Delta \rho_n)} \]
- Since the correlation is hopefully almost a pure single mode (or maybe just a few), the spectrum should be ~ a delta function at the mode.
- Thus, we can look for maxima in the spectrum
- There are better methods for calculating spectrum than a FFT sum – we are looking into
**GNSS TID Estimator**

- New Mexico Region
- All GPS stations within 500 km
- Chapman background profile
- Hooke TID
- Actual integration along slant paths
  - Real ephemeris for the day
  - 1 minute time intervals
  - 2 hours of simulation

**Results**

- Ran this case for a TID simulation of:
  - 300 km in longitude 500 km in latitude, 15 minute period
  - 75 km scale height 10 m/s at 120 km altitude
  - Saturated it at ~350 km
  - So a large easy to see wave.
- Upper right – example of simulated TID
- middle right – example of filtered TEC from two receivers
- Figure on lower right is spectra for PRN 30
Try methodology on Real GPS Data

- January 19, 2014 White Sands NM
- ~147 Standard GPS stations
- 15:75-17:75 UT

New issues with actual data
- Observations can have gaps in the data
- Can have cycle slips
- Need to perform QC
  - Minimum acceptable signal strength (0 or > 6)
  - Los of lock flag mod 2 = 0
- Need to have long enough continuous time series to filter, see full periods of waves, and compute correlations

Sample case with ~57 receivers preliminary analysis
Analysis of PRN 14

Mean Frequency: 5.83E-4 Hz
Mean Period: 1714 Seconds

Kx = 0.0135
Ky = 0.0635
X-(longitude) wavelength: 465 km
Y-(latitude) wavelength: 99 km

Direction: South and West
Hooke TEC again

- When estimating periods from GPS data, we have to take account of satellite motion
- Standard: ionospheric pierce point (IPP) for the GPS motion through the ionosphere
- This clearly still provides an error but how big and what does it depend on?
- If we start with a Hooke model of TIDS

\[
\delta T(x_r, y_r, \theta_r(t), \alpha_r(t), t) = \left(\frac{u_b \sin(I)}{\omega}\right) \text{Re} \left[ \langle N(t) \rangle e^{i\left(\omega(t-t_0) - k_x x_r - k_y y_r + \frac{\pi}{2} - \phi_0\right)} \right] W_S(p, t)
\]

\[
\langle N \rangle = \frac{1}{\sin(\theta_r)} \int \frac{N(z)}{L_N(z)} E(z) e^{-i\left[\left(k_x \sin(\alpha_r) + k_y \cos(\alpha_r)\right) \cot(\theta_r) + k_z\right] z + \Phi_N(z)}
\]

\[
E(z) = e^{\frac{(z-z_*)}{2H}} (z \leq z_T) = e^{\frac{-(z-z_T)}{2H}} (z > z_T)
\]

Space-time correlations:

\[ |\delta T(x_1, y_1, \theta_1(t), \alpha_1(t), t) \delta T(x_2, y_2, \theta_2(t'), \alpha_2(t'), t')| \]

\[ = \left(\frac{u_b \sin(I)}{\omega}\right)^2 \left\{ \langle |N_{1,2}(t, \tau)| \rangle \exp\left[ i \left(\omega \tau_{1,2} - k_x \Delta x_{1,2} - k_y \Delta y_{1,2}\right) \right] \right\} \]

\[ \langle |N_{1,2}(t, \tau)| \rangle = \int \, dz \, F(z) \int \, d\Delta z \, F(z + \Delta z) \left( e^{-i\left[ k_x \delta \hat{x}_{1,2}(t, \tau) + k_y \delta \hat{y}_{1,2}(t, \tau)\right] z} \right) e^{-i\left[ k_x \delta \hat{x}_{2}(t+\tau) + k_y \delta \hat{y}_{2}(t+\tau)\right] \Delta z} \right\}_t \]
Use of Hooke TEC to Estimate Frequency

- If we ignore the explicit frequency (omega) in the TEC model
  - Then the variation in time is ONLY DUE TO THE SATELLITE MOTION
  - Further, that variation has no approximation due to an IPP height
- IF we consider the satellite motion and the frequency motion as two function then we can write
  \[ F(\omega) = \int d\omega' G(\omega')H(\omega - \omega') \]
  \[ G(\omega) \approx \delta(\omega' - \omega_*) \]
  \[ F(\omega) \approx H(\omega - \omega_*) \]
- Where, G represents the F.T. of the pure wave, and H is the F.T of the satellite motion effect. Since the pure wave is a delta function in frequency space, we can get that the entire F.T should at omega should be equal to the satellite motion at omega – the “true frequency”
- Thus we can compute F from GPS delta TEC data.
- We can then compute H from the Hooke TEC model with zero frequency
- We can compute the maximums of each, difference the frequency and get the intrinsic frequency WITHOUT ANY IPP approximation
Results

- 20 variations in Hooke TEC altitudes
  - Vary 5 scale heights
    \[ HD = (30, 50, 77, 100, 125) \]
  - Vary 4 height maximums \[ ZT = (250, 300, 400, 500) \]
- Satellite motion frequency more than doubles
- Mainly dependent on saturation altitude
- Frequency variation is right in the 20-40 minute period
- Large effect!!!
**Conclusion**

- **Issue:**
  - The satellite motion frequency varies A LOT based on the exact vertical profile shape of the TID
  - The same effect occurs for estimating spatial spectrum or velocities, or any combination
  - There is no correct IPP point to take, no preferred altitude.
  - The finite thickness of the TIDS – which can extend well over 100 km or so creates an error for any kind of 2D correlation method.
  - Effect is minimized for shorter period / longer wavelength / faster speed waves

- **Solution/Way Forward**
  - Use GNSS satellites at GEO – no satellite motion
  - Use very high elevations
  - Have to know the vertical distribution
    - If known, possible to iteratively improve estimation in 2D
  - Direct 3D+time imaging of TIDS using GPS
  - Other data sets to help with vertical distribution
  - Parameterization and estimation
Summary

- We have modeled that analytical form of slant TEC from a Hooke model of TIDS
- We have shown the importance of satellite motion upon the TID estimation process
- To overcome limitations of closely clustered receivers and 2D correlations we have
  - Used as many GNSS receivers as possible over ~ 500 km baselines around the region of interest
  - introduced a spectral estimation process for the horizontal wavenumbers, periods, velocity
  - Demonstrated on simulated data
  - Estimated parameters on actual data from New Mexico
- Despite the generalization, and removal of some limitations of the new method we find:
  - The satellite motion produces a significant intrinsic error that cannot be removed by an 2D processing / analysis method
    - The 3D extended nature of the TID combined with the satellite motion produces an error that cannot be removed
    - The effect is worst for velocities ~ the GPS ionospheric speed
    - But always there
- The satellite motion in GPS combined with non-linear propagation in HF implies that we should not expect an apples to apples comparison – it is premature to say GPS TIDS and HF bottomside TIDS do not see same waves
- Full 3D methods need to be developed