Expanding the Frequency Resolution of TOA Analysis Applied to ELF/VLF Wave Generation Experiments at HAARP

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Abstract

Modulated HF heating of the ionosphere in the presence of natural ionospheric current sources has been used to generate electromagnetic ELF/VLF waves since the 1970's. In the ~1-5 kHz band, the amplitude and phase of the received ELF/VLF signal depends on the amplitude and phase of the conductivity modulation generated throughout the HF-heated ionospheric body, as well as on the signal propagation parameters (i.e., the attenuation and phase constants) between each of the current sources and the receiver. Previous signal processing advances have produced an accurate ELF/VLF time-of-arrival (TOA) analysis technique that differentiates line-of-sight and ionospherically-reflected signal components, allowing accurate calculation of the amplitude and phase of each component observed at the receiver. This TOA method requires a wide bandwidth (~2.5 kHz) in order to have accurate time resolution and therefore is relatively insensitive to the frequency-dependent nature of ELF/VLF wave propagation. In this paper, we present an improved ELF/VLF TOA method that is capable of providing high frequency resolution. The new analysis technique is applied to experimental observations of ELF/VLF signals generated by modulated heating at HAARP. We present measurements of the amplitude and phase of the received ELF/VLF signal as a function of frequency and compare the results with the predictions of an HF heating model.
**Introduction**

Time of arrival analysis [1] has been successfully applied to ELF/VLF waves generated by modulated HF heating of the ionosphere [2-5]. The method applies Fourier analysis to the received ELF/VLF signal and uses the known frequency-time chirp modulation format as input in order to determine the amplitude and phase of ELF/VLF signals incident upon a receiving antenna as a function of time (as shown in Figure 1). The time resolution of the method is determined by the reciprocal of the modulation bandwidth. Due to the large bandwidth requirements, past implementations of this method cannot be used to analyze the frequency dependence of the received signal. In this paper, we improve upon the method by using a second order Taylor series expansion of the system impulse response to approximate the amplitude and phase as a function of time and frequency. This new analysis technique is applied to experimental observations performed at HAARP and compared with the predictions of a theoretical model.
Figure 2. The modified TOA method iteratively zeros out discrete frequency ranges from the received data set, as depicted in the cartoon, providing the ability to evaluate frequency dependent wave generation and propagation effects.

Description of the Analysis Technique

For standard TOA analysis [1], the ELF/VLF modulation imposed upon the HF carrier is a linear frequency-time chirp that varies linearly from 1 to 5 kHz over 4 seconds. The received ELF/VLF waveform is mixed down to baseband using the expected frequency-time format kernel and low-pass filtered to reduce the effects of noise. The signal is then reconstituted by mixing the result back to the original frequency range (by mixing with the complex conjugate of the original mix-down kernel). The Fourier transform of this signal is then divided by the Fourier transform of the transmitted chirp modulation, resulting in a transfer function. The inverse Fourier transform is then applied, resulting in the impulse response of the system. Due to square windowing, this impulse response consists of a train of impulses (whose amplitudes and phases represent the amplitudes and phases of the signals incident upon the receiving antenna) convolved with a complex-valued sinc function. Standard TOA analysis applies a deconvolution method to distinguish discrete chirps separated in time by the reciprocal of the bandwidth employed.
High-frequency resolution can be supplied by re-interpreting the TOA data set. Individual frequency ranges are zeroed out and the deconvolution process is repeated. The resulting transfer function may be expressed:

\[
h(t) = \sum_{n=0}^{f_{\text{min}}} A_n e^{i\phi_n} e^{2\pi f_c(t-\tau_n)} \text{sinc}(BW(t-\tau_n)) + \sum_{n=f_{\text{max}}}^{5\text{ kHz}} A_n e^{i\phi_n} e^{2\pi f_c(t-\tau_n)} \text{sinc}(BW(t-\tau_n))
\]

If the zeroed frequency range is small enough, only slight differences in the amplitude, phase, and time of arrival are expected. Signifying the original analysis results with subscript T (for total), and signifying the differential analysis with subscript D (for differential), the following equation for the frequency-dependent terms (signified with subscript F) can be formed using a second-order expansion for the sinc function:

\[
\frac{A_T e^{i\phi_T} [1 + j2\pi f_c(t-\tau_T) - (2\pi^2 f_c^2 + \pi^2 BW^2)(t-\tau_T)^2]}{A_T e^{i\phi_T} [1 + j2\pi f_c(t-\tau_T) - (2\pi^2 f_c^2 + \pi^2 BW^2)(t-\tau_T)^2] - A_D e^{i\phi_D}}
\]

This equation results in a quadratic formula for the propagation delay, and using this result, the amplitude and phase as a function of frequency may be calculated as:

\[
A_T e^{i\phi_T} = \frac{A_T e^{i\phi_T} [1 + j2\pi f_c(\tau_D - \tau_T) - (2\pi^2 f_c^2 + \pi^2 BW^2)(\tau_D - \tau_T)^2] - A_D e^{i\phi_D}}{[1 + j2\pi f_c(\tau_D - \tau_T) - (2\pi^2 f_c^2 + \pi^2 BW^2)(\tau_D - \tau_T)^2]}
\]

Using these equations, we calculate the propagation delay, amplitude, and phase as a function of frequency for a given ELF/VLF observation.
**Figure 3.** A spectrogram of ELF/VLF waves generated by modulated heating of the ionosphere and observed at Paradise. Eight higher-order harmonics are observed in the data set. In this paper, we focus on the fundamental (1-5 kHz) chirp.

**Application to Experimental Observations**

The experiment related in this work was conducted at the HAARP Observatory in Gakona, Alaska. The HF beam was directed at 15° off-zenith and at 81° azimuth, towards the receiver at Paradise (~100 km distant). The HF transmission was at 3.25 MHz (X-mode), and the beam was square wave amplitude modulated at 100% depth with a linear frequency-time ramp (chirp) from 1-5 kHz over four seconds. The receiver used is composed of 2 orthogonal magnetic loop antennas and sampled at 100 kHz, synchronized to GPS. It is located approximately 100 km away from the HAARP facility.

Spectrogram format observations are presented in Figure 3. The transmitted frequency-time chirp is clearly visible (together with harmonics) in the spectrogram figure. Also present are short time duration vertical lines that are generated by impulsive lightning return strokes.

The results of the application of the new methodology to this data set are shown in Figure 4. Amplitude, phase, and time-of-arrival (or propagation delay) are shown in three panels as a
function of frequency. The dependence on frequency is evident in all three panels. The comparison with theoretical predictions remains to be performed.

**Summary and Conclusions**

We summarize this paper with the following three points:

1) The high frequency resolution TOA analysis method produces results consistent with expectations below $\sim 3$ kHz.
2) The null observed near 3.5 kHz is likely a signal processing effect where two sinc functions interfered with the methodology applied the modified TOA analysis.

3) We plan to improve the modified TOA method by using higher-order Taylor series expansion approximations and investigating the role that ionosphere reflections play at higher modulation frequencies.

References