Multiple Phase Screen (MPS) Calculation of Two-way Spherical Wave Propagation in the Ionosphere

Ionospheric Effects Symposium

May 2015

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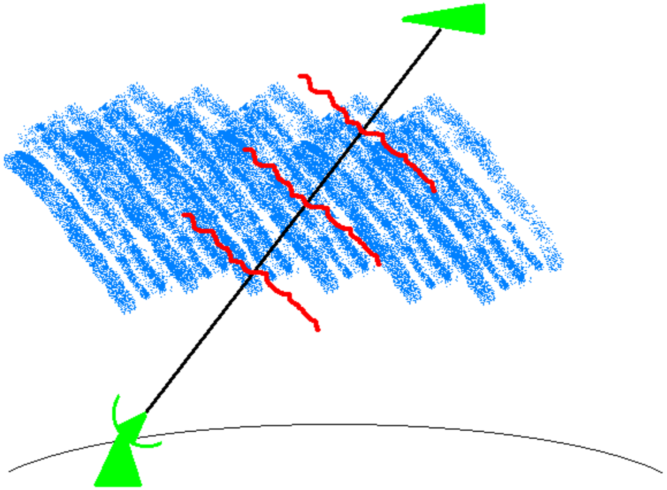
Outline

- Introduction
- Formulation of the solution
- Examples
  - Scintillation index for two-way propagation
    - Monostatic geometry
    - Bistatic geometry
  - Reciprocity
  - Two-way propagation with multiple correlated scatterers
- Conclusions

Introduction (1/2)
MPS Signal Generation



- Parabolic Wave Equation for E-field

$$\left(\frac{1}{z'^2}\right)\left(\frac{\partial^2 U}{\partial \theta^2}+\frac{\partial^2 U}{\partial \phi^2}\right)-i2k\frac{\partial U}{\partial z'}+k^2\beta\epsilon U=0$$

Diffraction term

Source term

Solution Method:

- **Collapse ionospheric structure to multiple thin phase-changing screens with free space between**
- **At phase screen, neglect diffraction term**
- **Between screens, the PWE is source free, so can solve by Fourier Transform method**
- **Solution U is the single-frequency transfer function. U is the Fourier transform of the impulse response function.**

Introduction (2/2)

- Impulse response function
  - Convolve the impulse response function with the transmitted waveform to obtain the received, disturbed waveform
- Two methods to calculate the impulse response function:
  - Statistical techniques:
    - Techniques based on the mutual coherence function (MCF)
    - Starting point is the analytic solution for the two-frequency, two-time, two-position MCF (the correlation function of the propagating electric field)
    - Theoretical calculation requires strong scattering, S4 equal to unity, phase structure function must be quadratic, signal bandwidth is small, structure is homogeneous.
    - Limitations never fully studied
    - Previously the choice for most receiver testing because of speed and relative simplicity. But, still in use now for strategic systems
  - Multiple phase screen (MPS) techniques
    - Most accurate technique available. Starting point is a realization of the in-situ electron density. None of the limitations above apply.

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Formulation

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Scalar Helmholtz equation

$$\nabla^2\psi + k^2(1+\beta\epsilon)\psi = 0$$

where

$$\beta = \frac{-\omega_p^2}{\omega^2 - \omega_p^2}$$

$$\epsilon = \frac{\Delta N_e}{\langle N_e \rangle}$$

$$\omega_p^2 = 4\pi c^2 r_e \langle N_e \rangle$$

$$k = \frac{2\pi}{\lambda}$$

Formulation

Substitute the parabolic approximation for a spherical wave

$$\psi(x,y,z) = \frac{U(x,y,z)}{(z-z_t)} \times \exp\left\{-ik\left(z - z_t + \frac{(x-x_t)^2 + (y-y_t)^2}{2(z-z_t)}\right)\right\}$$

Make the substitutions

$$\theta = \frac{(x-x_t)}{z'}; \qquad \phi = \frac{(y-y_t)}{z'}; \qquad z' = z - z_t$$

To obtain the final parabolic wave equation (PWE)

$$\left(\frac{1}{z'^2}\right)\left(\frac{\partial^2 U}{\partial\theta^2} + \frac{\partial^2 U}{\partial\phi^2}\right) - i2k\frac{\partial U}{\partial z'} + k^2\beta\epsilon U = 0$$

Propagation through a phase screen: solve PWE with diffraction term set to zero

$$U\left(\theta, \frac{\Delta z'}{2}\right) = U\left(\theta, \frac{-\Delta z'}{2}\right)\exp\left\{-ik\int_{\frac{-\Delta z'}{2}}^{\frac{\Delta z'}{2}} \Delta n(\theta,\zeta)\, d\zeta\right\}$$

Formulation

Free-space propagation between phase screens: set source term to zero and solve remaining equation via FFTs

$$\begin{aligned} U(\theta, z') &= \int_{-\infty}^{\infty} \hat{U}(q_\theta, z') \exp\left(i2\pi q_\theta \theta\right) dq_\theta \\ \hat{U}(q_\theta, z') &= \int_{-\infty}^{\infty} U(\theta, z') \exp\left(-i2\pi q_\theta \theta\right) d\theta \end{aligned}$$

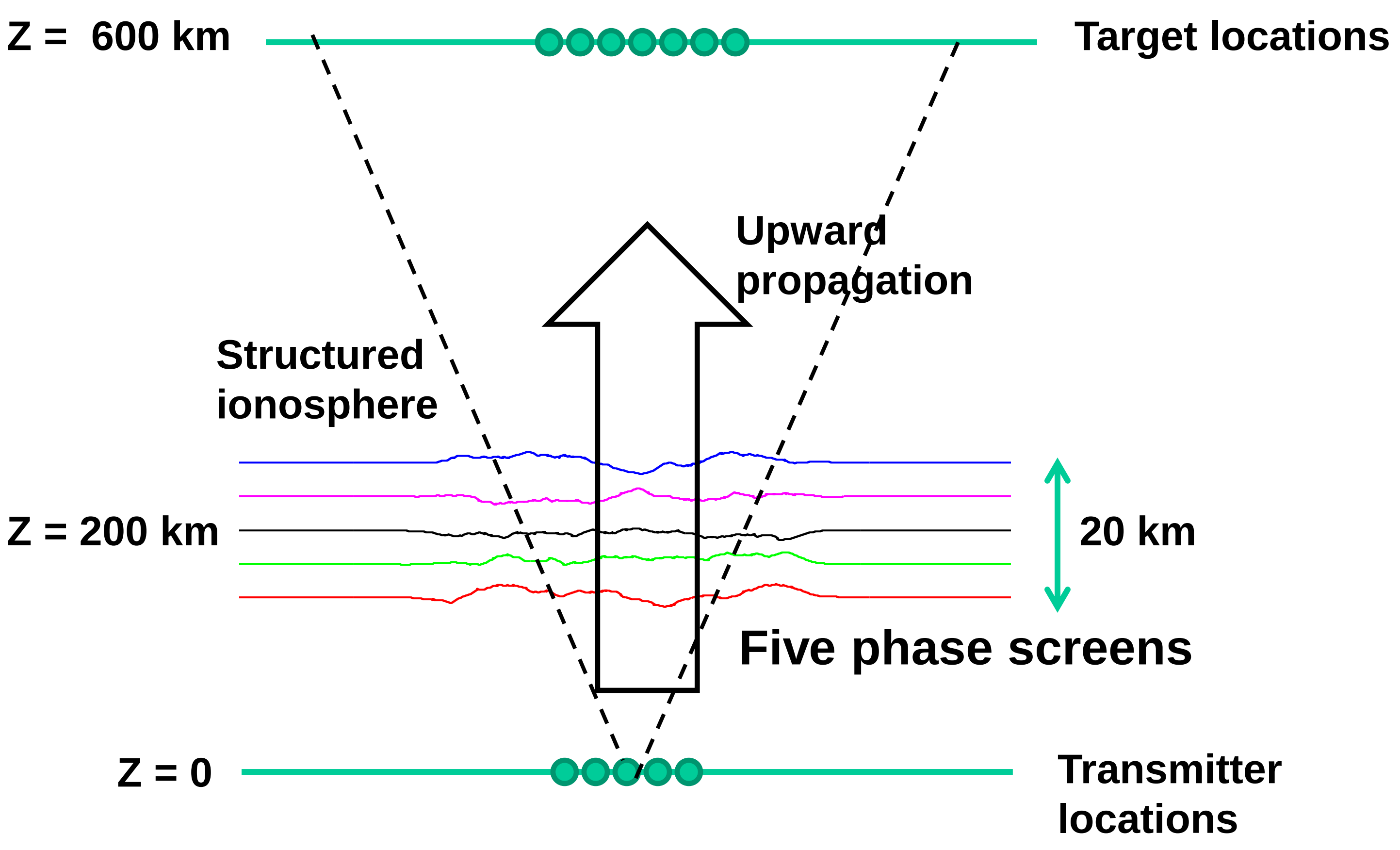
The solution for free-space propagation is

$$\begin{aligned} U(\theta, z_2') &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U(\theta', z_1') \exp\left\{\frac{i2\pi^2 q_\theta^2}{k}\left(\frac{1}{z_1'} - \frac{1}{z_2'}\right)\right\} \exp\left(i2\pi q_\theta(\theta - \theta')\right) dq_\theta' \, d\theta' \\ &= \sqrt{\frac{iz^*}{\lambda}} \int_{-\infty}^{\infty} U(\theta', z_1') \exp\left\{\frac{-ikz^*}{2}(\theta - \theta')^2\right\} d\theta' \end{aligned}$$

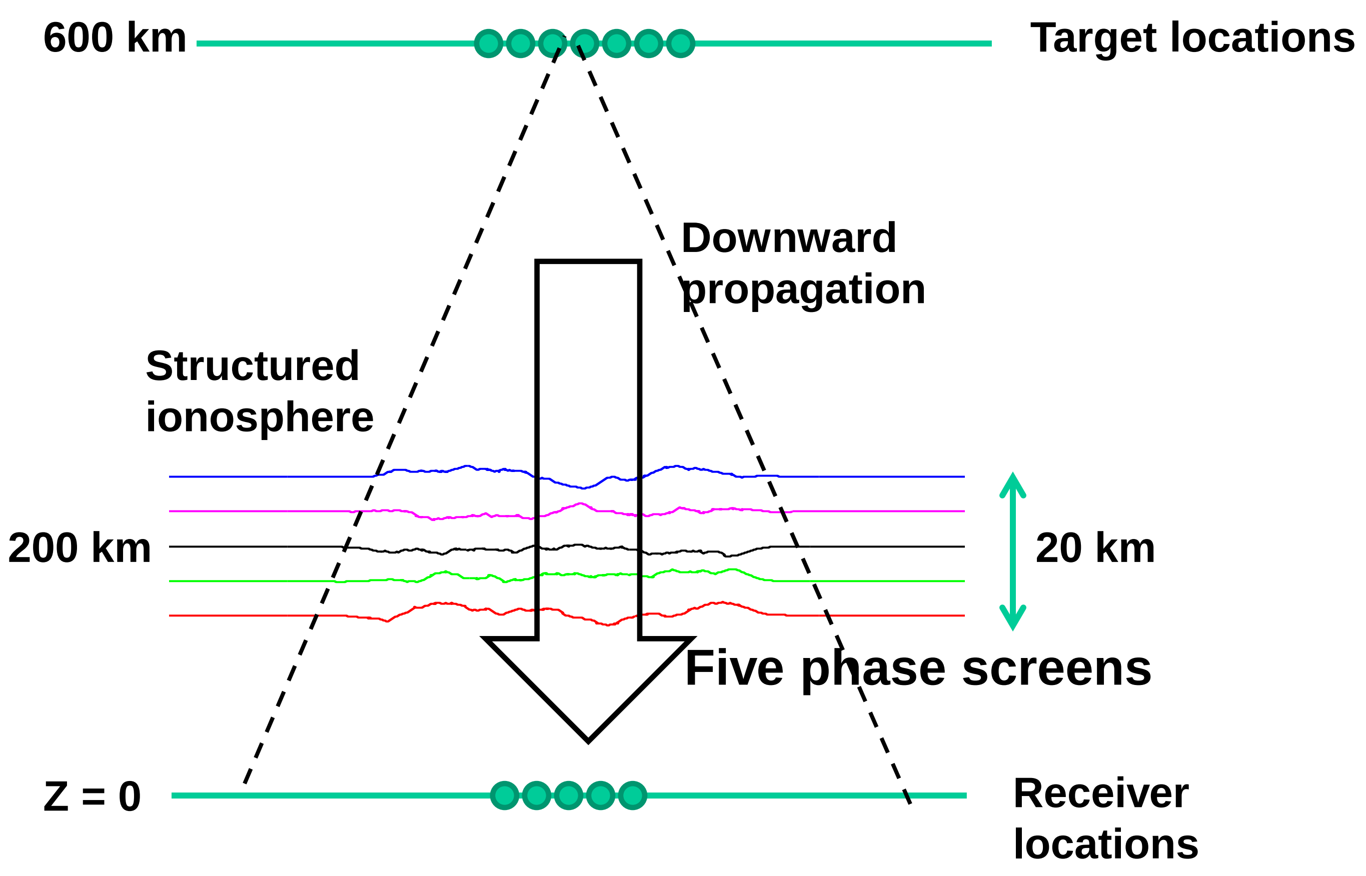
where

$$\frac{1}{z^*} = \left(\frac{1}{z_1'} - \frac{1}{z_2'}\right)$$

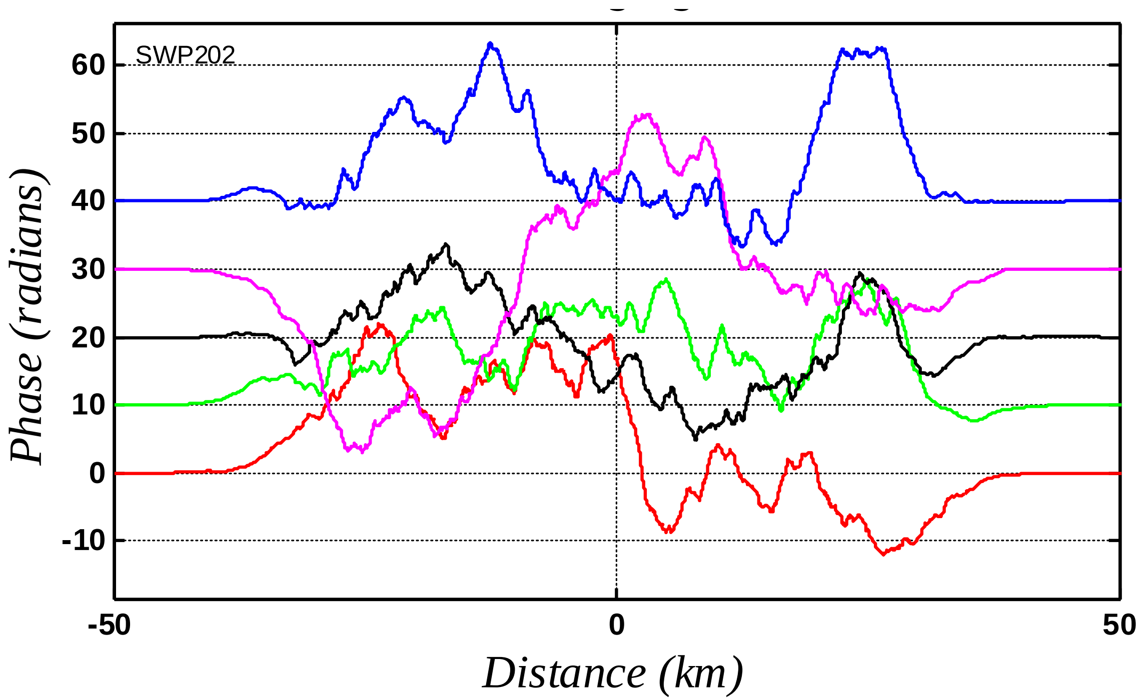
Propagation Geometry Used in the Following Examples



Propagation Geometry Used in the Examples

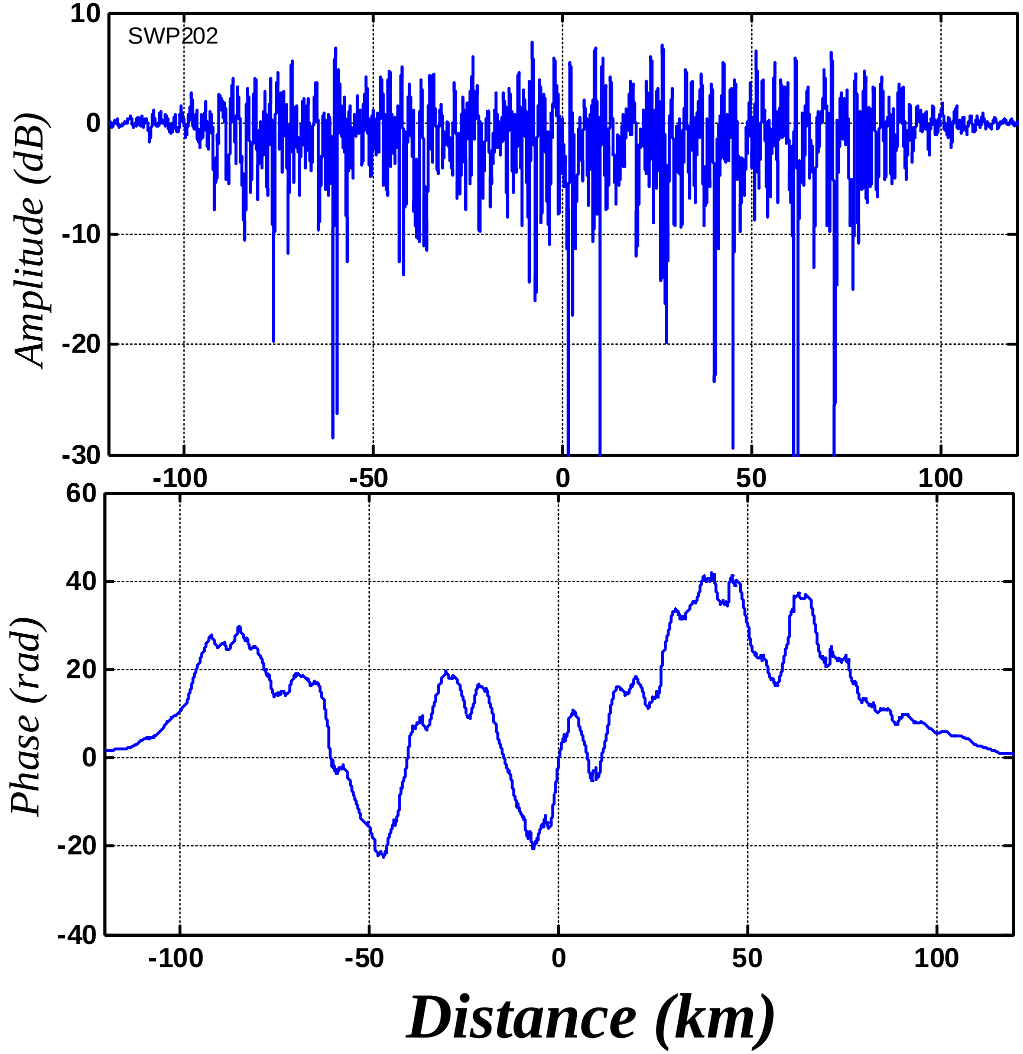


Close-up of Five Phase Screens



- Values of phase shown are separated by 10 radians
- Screens extend in altitude from 190 to 210 km
- Length of phase screen at 200 km altitude is 200 km
- Phase screens are generated to have a K^{-3} PSD, outer scale of 5 km, inner scale of 10 m, and are comprised of 2^{19} points.

Electric Field in the Target Plane Due to a Single Transmitter



Electric field at z = 600 km caused by a single element located at z = 0, after propagation through five phase screens

Two-way Value of the Scintillation Index

Definition of the S4 scintillation index, the normalized standard deviation of the received power

$$S_4^2(\text{one-way}) = \frac{\langle (s - \langle s \rangle)^2 \rangle}{\langle s \rangle^2}$$

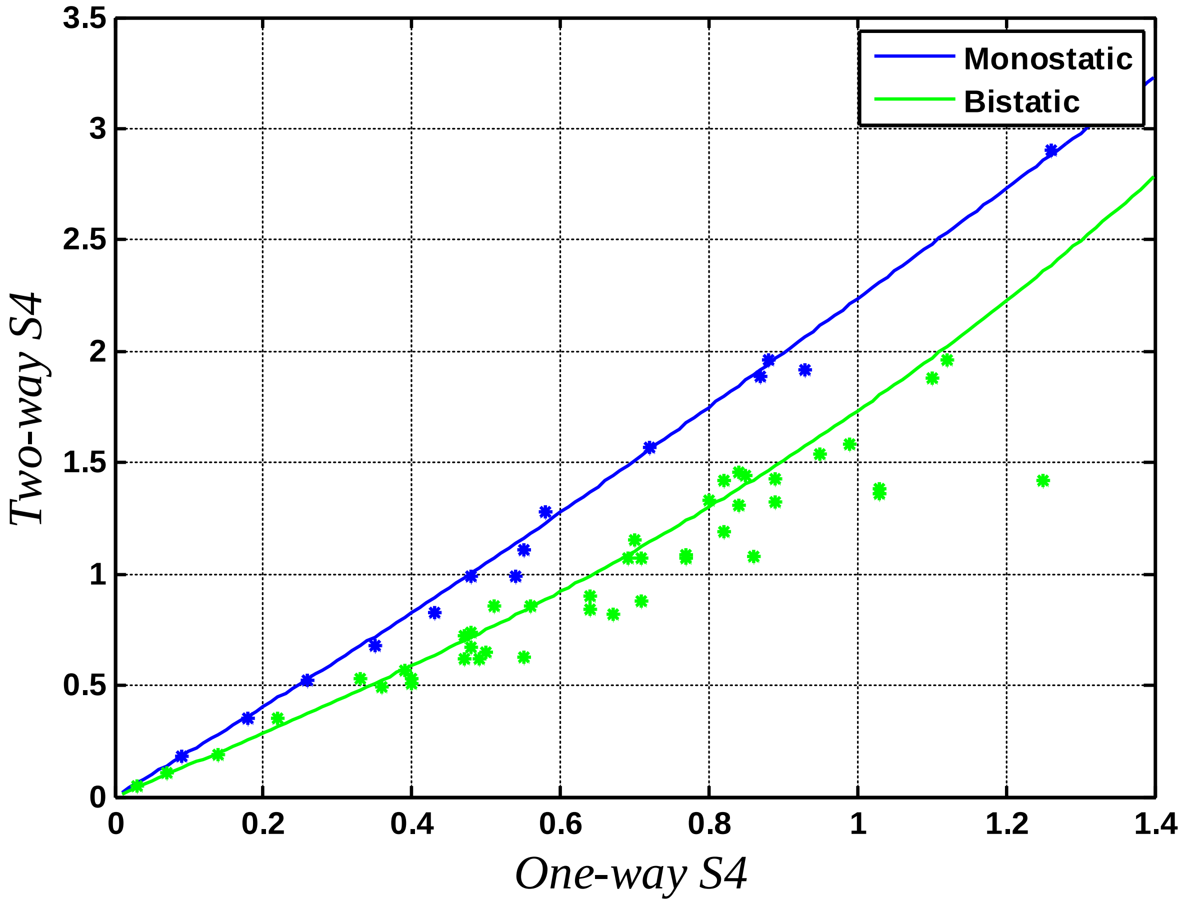
For monostatic (radar) two-way propagation

$$S_4^2(\text{monostatic}) = S_4^2(\text{one-way}) \left(\frac{4 + 6S_4^2(\text{one-way})}{1 + S_4^2(\text{one-way})} \right)$$

For bistatic two-way propagation with independent up and down paths

$$S_4^2(\text{bistatic}) = S_4^2(\text{up}) \left(2 + S_4^2(\text{up}) \right)$$

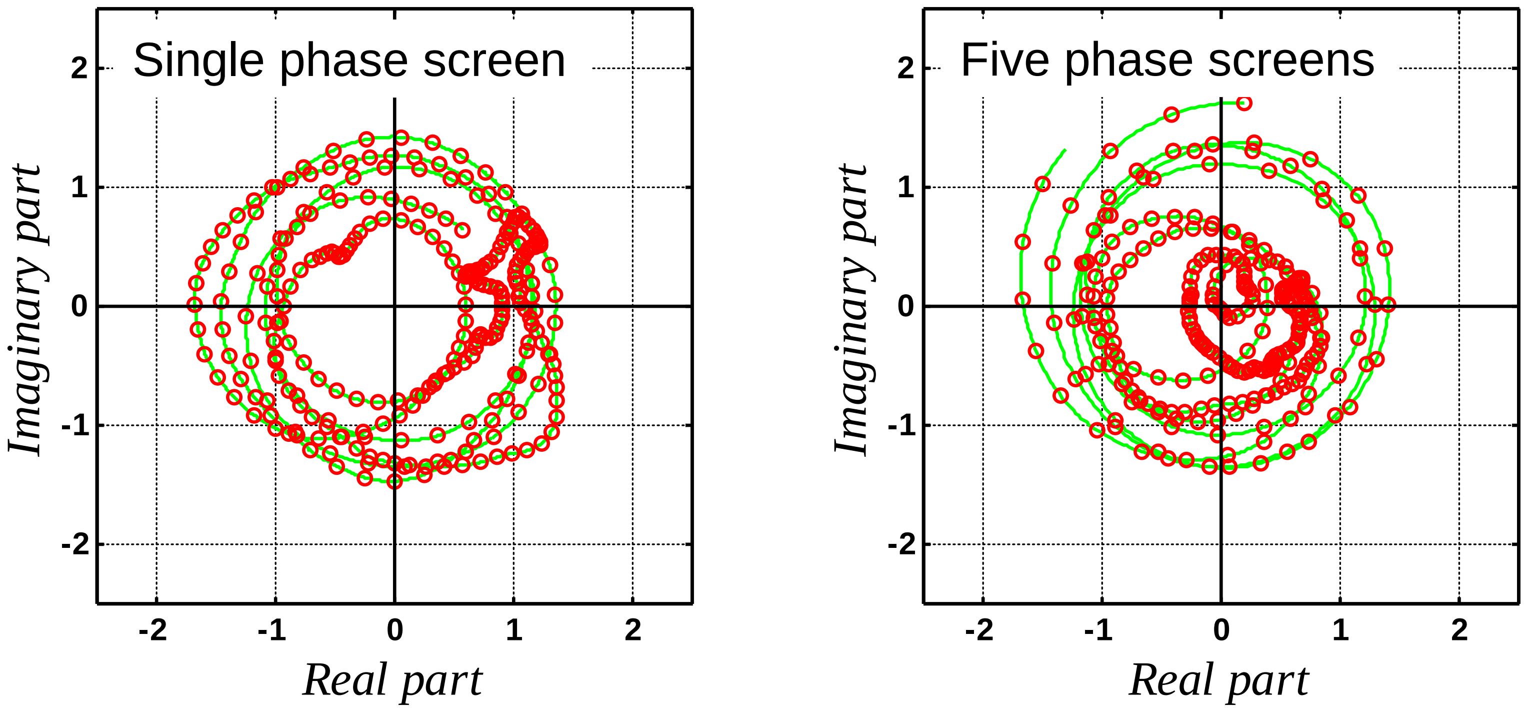
Scintillation Index for Two-way Propagation



Theory: solid lines; Simulation: dots

Radar detection performance is a strong function of S_4

Reciprocity is Satisfied



- **Reciprocity: Field is same if transmitter and receiver are interchanged.**
- **The figures show I/Q plots of the complex one-way field comparing upward (green curve) and downward (red circles) propagation**
- **Upward propagation from single transmitter to many receive locations. Downward propagation from original receive locations to the single original transmitter location**

Two-way Propagation

Field at target plane due to many transmitter elements

$$V(x, z_{tar}) = \sum_{xmtr} V(x_{xmtr}, z_{xmtr}) G_{up}(xmtr \rightarrow tar)$$

Field at receiver plane due to scatterers in target plane

$$\begin{aligned} V(x, z_{rcvr}) &= \sum_{tar} V(x, z_{tar}) G_{down}(tar \rightarrow rcvr) \\ &= \sum_{xmtr} V(x_{xmtr}, z_{xmtr}) \sum_{tar} G_{up}(xmtr \rightarrow tar) G_{down}(tar \rightarrow rcvr) \end{aligned}$$

Following two examples of two-way propagation:

One transmitter at center of MPS grid

Upward propagation through five phase screens

401 target scatterers at z = 600 km, spaced by λ/2

Downward propagation back to receiver plane

Two-way Propagation, Weak Scattering, Linear Group of Scatterers

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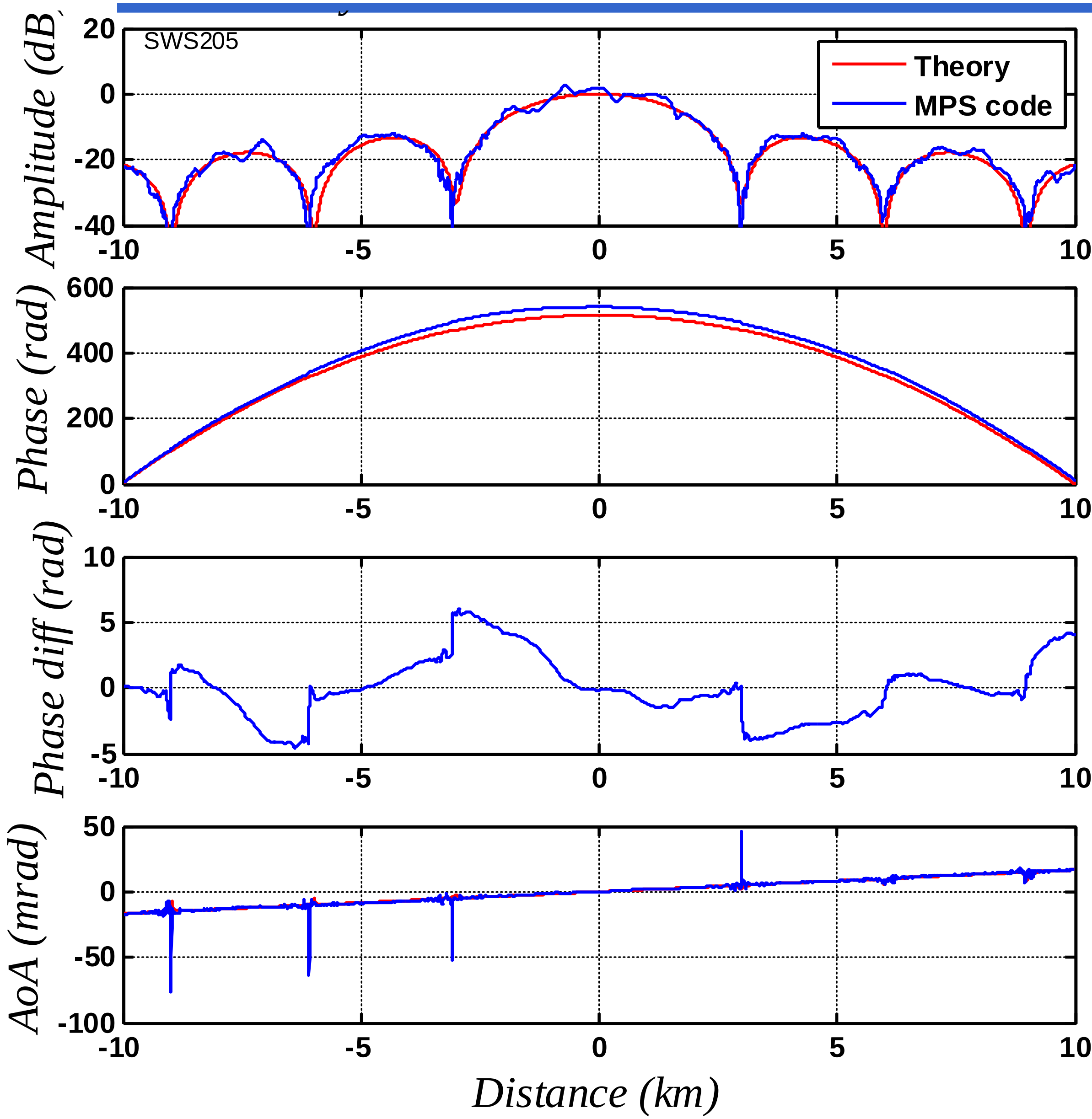
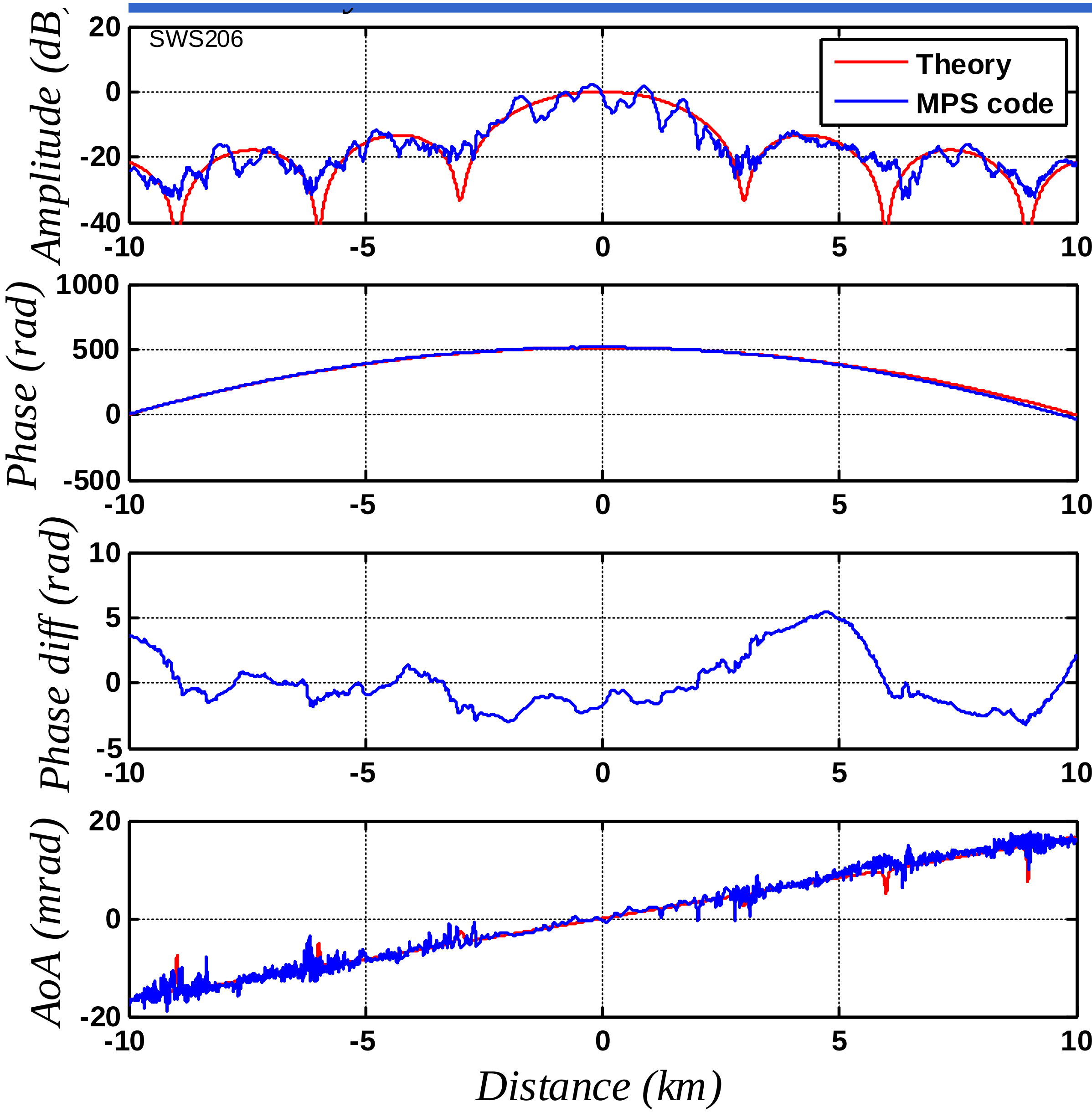


- 5 screens near z = 200 km
- 401 scatterers at z = 600 km
- S4(one-way) = 0.16
- Figure shows small portion of MPS grid
- Smooth red curve is theory for case of no scintillation
- Blue is MPS result
- Measurement of AoA uses 10-m antenna & correlation technique

Two-way Propagation, Stronger Scattering, Linear Group of Scatterers



- 5 screens near z = 200 km
- 401 scatterers at z = 600 km
- S4(one-way) = 0.46
- Figure shows small portion of MPS grid
- Smooth red curve is theory for case of no scintillation
- Blue is MPS result
- Measurement of AoA uses 10-m antenna & correlation technique

Conclusions

- **Originally developed for application to synthetic aperture radar**
- **Includes the correlation of signals propagating on closely-spaced paths**
- **Avoids the small-scene approximation**
- **Code design allows for variation in RCS of the target scatterers**
- **Additional but straightforward work needed for:**
  - **3D propagation**
  - **Application to wide bandwidth waveforms**